NAG Toolbox for MATLAB

f08ce

1 Purpose

f08ce computes a QL factorization of a real m by n matrix A.

2 Syntax

```
[a, tau, info] = f08ce(a, 'm', m, 'n', n)
```

3 Description

f08ce forms the QL factorization of an arbitrary rectangular real m by n matrix.

If $m \geq n$, the factorization is given by:

$$A = Q\begin{pmatrix} 0 \\ L \end{pmatrix},$$

where L is an n by n lower triangular matrix and Q is an m by m orthogonal matrix. If $m < n$ the factorization is given by

$$A = QL,$$

where L is an m by n lower trapezoidal matrix and Q is again an m by m orthogonal matrix. In the case where $m > n$ the factorization can be expressed as

$$A = \begin{pmatrix} Q_1 & Q_2 \end{pmatrix}\begin{pmatrix} 0 \\ L \end{pmatrix} = Q_2 L,$$

where Q_1 consists of the first $m - n$ columns of Q, and Q_2 the remaining n columns.

The matrix Q is not formed explicitly but is represented as a product of $\min(m,n)$ elementary reflectors (see Section 3.2.6 in the F08 Chapter Introduction for details). Functions are provided to work with Q in this representation (see Section 8).

Note also that for any $k < n$, the information returned in the last k columns of the array **a** represents a QL factorization of the last k columns of the original matrix A.

4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D 1999 *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia URL: http://www.netlib.org/lapack/lug

Golub G H and Van Loan C F 1996 *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **a(lda,$*$) – double array**

The first dimension of the array **a** must be at least $\max(1, \mathbf{m})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

The m by n matrix A.

5.2 Optional Input Parameters

1: **m – int32 scalar**

Default: The first dimension of the array **a**.

m, the number of rows of the matrix A.

Constraint: $\mathbf{m} \geq 0$.

2: **n – int32 scalar**

Default: The second dimension of the array **a**.

n, the number of columns of the matrix A.

Constraint: $\mathbf{n} \geq 0$.

5.3 Input Parameters Omitted from the MATLAB Interface

lda, work, lwork

5.4 Output Parameters

1: **a(lda,$*$) – double array**

The first dimension of the array **a** must be at least $\max(1, \mathbf{m})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

If $m \geq n$, the lower triangle of the subarray $\mathbf{a}(m - n + 1 : m, 1 : n)$ contains the n by n lower triangular matrix L.

If $m \leq n$, the elements on and below the $(n - m)$th superdiagonal contain the m by n lower trapezoidal matrix L. The remaining elements, with the array **tau**, represent the orthogonal matrix Q as a product of elementary reflectors (see Section 3.2.6 in the F08 Chapter Introduction).

2: **tau($*$) – double array**

Note: the dimension of the array **tau** must be at least $\max(1, \min(\mathbf{m}, \mathbf{n}))$.

The scalar factors of the elementary reflectors (see Section 8).

3: **info – int32 scalar**

$\mathbf{info} = 0$ unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

$\mathbf{info} = -i$

If $\mathbf{info} = -i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **m**, 2: **n**, 3: **a**, 4: **lda**, 5: **tau**, 6: **work**, 7: **lwork**, 8: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

7 Accuracy

The computed factorization is the exact factorization of a nearby matrix $(A+E)$, where

$$\|E\|_2 = O(\epsilon)\|A\|_2,$$

and ϵ is the ***machine precision***.

8 Further Comments

The total number of floating-point operations is approximately $\frac{2}{3}n^2(3m-n)$ if $m \geq n$ or $\frac{2}{3}m^2(3n-m)$ if $m < n$.

To form the orthogonal matrix Q f08ce may be followed by a call to f08cf:

```
[a, info] = f08cf(a(:,1:m), tau);
```

but note that the second dimension of the array **a** must be at least **m**, which may be larger than was required by f08ce.

When $m \geq n$, it is often only the first n columns of Q that are required, and they may be formed by the call:

```
[a, info] = f08cf(a, tau);
```

To apply Q to an arbitrary real rectangular matrix C, f08ce may be followed by a call to f08cg. For example,

```
[c, info] = f08cg('Left','Transpose', a(:,1:min(m,n)), tau, c);
```

forms $C = Q^{T}C$, where C is m by p.

The complex analogue of this function is f08cs.

9 Example

```
a = [-0.57, -1.28, -0.39, 0.25;
     -1.93, 1.08, -0.31, -2.14;
     2.3, 0.24, 0.4, -0.35;
     -1.93, 0.64, -0.66, 0.08;
     0.15, 0.3, 0.15, -2.13;
     -0.02, 1.03, -1.43, 0.5];
[aOut, tau, info] = f08ce(a)
```

```
aOut =
   -0.2537   -0.7752    0.1095    0.0696
   -0.0957    0.2276    0.5122   -0.5958
   -2.8948    0.2084   -0.0952   -0.0975
   -0.5041   -1.5813    0.2476    0.0223
    1.9213   -1.0532    1.6928   -0.5931
   -0.8730    0.9018    0.2139   -3.0916
tau =
    1.8631
    1.1791
    1.4873
    1.1617
info =
           0
```